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EMBEDDINGS OF SPACES WITH MULTIWEIGHTED DERIVATIVES AND THEIR APPLICATIONS

ABSTRACT

of the dissertation for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100 – Mathematics

The structure and scope of the thesis. The dissertation work consists of an introduction, three sections (each section consists of paragraphs), a conclusion and a list of used literature. The total number of pages is 94.

The number of sources used is - 65.

Keywords. Weight function, the norm of space, equivalent norms, multiweighted derivatives, multiweighted space, boundary value of a function, weight inequality, locally absolute continuous function.

Relevance of the research topic. The dissertation is devoted to the study of embeddings of spaces with multiweighted derivatives and their applications.

In the first half of the twentieth century, in accordance with the requirements of production, in the mathematical modeling of various phenomena and processes, singular differential equations appeared, and the problem of setting boundary value problems for these equations arose. Therefore, the study of spaces of differentiable functions with weights began. In these studies, according to changes in the vicinity of the boundaries of the coefficients of singular differential equations, they were called weakly singular and strictly singular equations. If the equation is weakly singular in the neighborhood of the boundary, then the problem is posed on the boundary according to regular equations. And if the equation is strictly singular in the neighborhood of the boundary, then the problem is not posed at the boundary. But, if in the neighborhood of the border a strictly singular equation describes a model of some phenomenon in life, then this phenomenon must obey one physical law, that is, there must be one "boundary" problem at the border. Therefore, L.D. Kudryavtsev considered the space of differentiable functions of one variable on the semiaxis, in which the difference between the function and the algebraic polynomial is zero when the argument tends to infinity and called the coefficients of this polynomial "boundary" values of the function at infinity.

In 1991-1992, at a scientific seminar under his supervision, R. Oinarov expanded the idea of L.D. Kudryavtsev and set the task of studying a multiweighted space $W^n_{p,\bar{\rho}}$, showing ways to solve boundary value problems at singular finite and infinite points in simple differential equations. Later this space was called the multiweighted derivatives space.

At the beginning, when the weights were power functions, that is $\rho_i = t^{\alpha_i}$, $i = 1,2,...,n$, numerous results were obtained by A.O. Bayarystanov, B.L. Baidel'dinov, A.A. Kalybay, Z.T. Abdykalykova. For example, B.L.Baidel'dinov showed how to represent a boundary value problem in the space $W^n_{p,\bar{\alpha}}$ depending on the singularity of a simple differential equation of symmetric order n.

A.A. Kalybay showed that the generalized Cauchy problem is well-posed for a simple differential equation with a singularity of the nth order, regardless of the singularity of the equation at zero.

This result was achieved due to the properties of space $W^n_{p,\bar{\alpha}}$. These works show that the space $W^n_{p,\bar{\rho}}$ has big potential. And the study of space with multiweighted derivatives as a whole is a relevant problem.

Therefore, in the dissertation work, expanding the proposal of L.D. Kudryavtsev, a space called the space of functions differentiable after each multiplied weight and multiweighted derivatives is considered, as well as the boundary values of the function at zero and at infinity are determined, and various properties of this space are investigated.

Goals:

-obtain necessary and sufficient conditions for any function of the space $W^n_{p,\bar{\rho}}$ to become stable with respect to a polynomial at zero and at infinity in terms of multiweights;

- define functionals with the coefficients of polynomials, equivalent to the norm of the space $W^n_{p,\bar{\rho}}$, when any function in space $W^n_{p,\bar{\rho}}$ becomes stable at zero and at infinity with respect to a polynomial;

- obtain theorems on continuous and compact embedding of two spaces with different metrics with different multiweighted derivatives into each other;

- describe the closure of the set of finite functions in the space $W^n_{p,\bar{\rho}}$, in terms of traces, depending on the behavior of the $\bar{\rho}$ - multiweighted at the ends of the interval;

- determine the conditions for the density of a set of compactly supported functions in space $W^n_{p,\bar{\rho}}$.

The object of research. Properties of space with multiweighted derivatives of functions, embedding theorems and their applications to the Nikolskii-Lizorkin-Kudryavtsev inequality, properties of the closure of the set of compactly supported functions in space.

Research method. The dissertation uses the methods of functional analysis, the theory of weighted function spaces, as well as the results of boundedness and compactness of integral operators.

Scientific novelty and practical value of the dissertation.

In this work, necessary and sufficient conditions are obtained in terms of the multiweights of any function in the space $W^n_{p,\bar{\rho}}$ becomes stable to a polynomial at zero and at infinity, and rate of becomes stable to the polynomial is estimated. Also,

when any function of the space $W_{p,\bar{\rho}}^{n}$ becomes stable at zero and at infinity with respect to a polynomial, functionals equivalent to the norm of the space $W_{p,\bar{\rho}}^{n}$ are defined through the coefficients of these polynomials.

Criteria are obtained for the continuous and compact embedding of two spaces with multiweighted derivatives when spaces have different weights in different metrics. Criteria are obtained for the existence of traces of functions in the space $W_{p,\bar{\rho}}^{n}$ at the ends of the interval are found. As a consequence, necessary and sufficient conditions were obtained for the density of the set of compactly supported functions in the space $W_{p,\bar{\rho}}^{n}$ and, in terms of traces of functions, the reverse case is described in detail when the set of compactly supported functions in the space $W_{p,\bar{\rho}}^{n}$ is not dense.

The work has a fundamental character, it can be applied in solving problems of the theory of setting boundary value problems for singular differential equations. The results obtained can be used to conduct special courses for students, undergraduates, and doctoral students.

Approbation of the dissertation results. The main results and scientific conclusions of the dissertation research were discussed at the scientific seminars "Functional analysis and their applications" of the department of "Fundamental mathematics" of the L.N. Gumilyov Eurasian National University and discussed at the following international conferences:

- international conference “Functional spaces. Differential operators. Problems of Mathematical Education” dedicated to the 95th anniversary of the birth of corresponding member of the Russian academy of sciences, academician of the European academy of sciences L.D. Kudryavtsev, Russia, Moscow, November 26-29, 2018;
- International conference "Problems of differential equations, analysis and algebra", Aktobe, November 1, 2018;
- International conference "Actual problems of analysis, differential equations and algebra" (EMJ-2019), dedicated to the 10th anniversary of the issue of the journal "Eurasian Mathematical Journal" (EMJ-2019), Nur-Sultan, October 16-19, 2019.

The main content of the dissertation. The introduction addresses the following issues: the relevance of the work, the main goal, the object and methods of research, the novelty and practical value of the dissertation, the approbation of the results obtained.

The first section defines a space with multiweighted derivatives and some of its properties. In addition, the behavior of the function at the boundary of the investigated space was investigated. In the second section, we establish a continuous and compact embedding between spaces with multiweighted derivatives, and, as an application of these results, we show that the inequality of the Nikol'skii-Lizorkin-Kudryavtsev type holds. In the third section, theorems on the density of functions with compact support in a space with multiweighted derivatives are proved, and the

properties of the closure of the set of functions with compact support in this space are determined.

Publications. The main results of the dissertation were published in 10 papers. Of these, 3 in journals recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan, 2 in the journal from the Scopus database with a CiteScore percentile of no less than 25, 5 in proceedings of international scientific conferences.